Goldrush Dynamics of Private Equity*

Samuel Lee[†] Daniel Sunesson[‡]

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Abstract

We present a simple dynamic model of entry and exit in a private equity market with heterogeneous fund managers, a depletable stock of target companies, and learning about investment profitability. Its predictions match a number of stylized facts: Aggregate fund activity follows waves with endogenous transitions from boom to bust. Supply and demand in the private equity market are inelastic, and supply comoves with investment valuations. High industry performance precedes high entry, which in turn precedes low industry performance. Differences in fund performance are persistent, first-time funds underperform the industry, and first-time funds raised in boom periods are unlikely to see follow-on activity. Fund performance and fund size are positively correlated across firms, but negatively correlated across consecutive funds by the same private equity firm. Finally, boom periods can make "too much capital chase too few deals."

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[†]Stockholm School of Economics, samuel.lee@hhs.se.

[‡]Stockholm School of Economics, daniel.sunesson@hhs.se.

1 Introduction

Capital commitments and investments in the private equity industry are cyclical. This has been documented for the venture capital industry by Gompers and Lerner (2000) and Lerner (2002), and for the buyout industry by Kaplan and Stein (1993) and Kaplan and Strömberg (2009). To give a recent example, the global buyout volume shrunk from around \$527 billion in early 2007 to around \$124 billion by mid-2008. Such boom-bust patterns suggest that the private equity business is transitory, expanding and contracting as the opportunities for profitable control investments emerge and disappear.

We develop a simple model which captures this transient nature. It produces waves which endogenously transition from booms to busts. Furthermore, the dynamics of entry, prices and returns *within* a wave match a wide range of empirical patterns: the inelasticity of private equity supply to private equity demand and vice versa; the procyclicality of capital inflow and investment valuations; persistent performance differences across private equity firms; the underperformance of first-time funds; the positive (negative) relationship between entry and past industry returns (subsequent industry returns); the positive (negative) relationship between fund performance and fund size in the cross-section (in the time-series); and the notion of "too much capital chasing too few deals."[1]

The basic idea behind the model is to liken private equity waves to goldrushes. A goldrush starts with the discovery of gold which attracts gold diggers who settle nearby in the hope of making a fortune. As more gold is extracted over time, more gold diggers migrate to the area until all claims are staked. When the gold reserves dry up, the gold diggers either retire or migrate to the next discovery.[2] Our model essentially draws an analogy between gold discoveries and the emergence of private equity investment opportunities, gold diggers and private equity firms, claims and investments, as well as gold and investment

[1] These empirical patterns are documented by Gompers and Lerner (1999, 2000), Kaplan and Schoar (2005), Acharya et al. (2007), and Hochberg et al. (2008). The reported performance patterns in private equity stands in stark contrast to the evidence in the mutual fund industry (Malkiel, 1995; Berk and Green, 2004) and the investment management industry (Busse et al., 2008).

[2] An example is the Klondike Goldrush. In August 1896, gold was discovered in the Klondike river. By the summer of 1897, the nearby town of Dawson had grown to a population of 3,500. Around that time, steamships unloaded about one and half million dollars worth of Klondike gold in San Francisco and Seattle. Within half a year, the population of Dawson climbed to over 30,000. In the summer of 1899, the goldrush was officially over.

returns.

In the model, a fixed population of companies becomes improvable because of a latent productivity shock. To keep matters simple, the improvement can only be realized by private equity firms, investment firms specialized in acquiring and reorganizing companies. To do so, a private equity firm must raise a private equity fund, find a target company, and negotiate a price at which the company's shareholders are willing to sell the company. There are many private equity firms that repeatedly decide whether to raise a fund to acquire a company, i.e, whether to enter the private equity market. Each firm's entry decision depends on its own management ability, the number of available target companies, and the expected gains from reorganization. Importantly, the true expected gains are unknown but can be partially inferred from past investment outcomes. This learning process creates a link between past and current entry decisions.

The model yields a private equity wave under the plausible assumption that—absent positive experiences—the market's (prior) expectations are low. In that case, only few private equity funds are raised at the outset. When the true shock is low, these early funds earn disappointing returns, and investment activity subsequently subsides. Conversely, when the true shock is high, the early funds earn promising returns, which attracts other private equity firms into the market. As fund activity rises, the magnitude of the shock is revealed at a faster rate, which in turn accelerates entry. This feedback loop between learning and entry fuels the boom. The countereffect is that the influx of new funds depletes the pool of target companies faster. The accelerating attrition ultimately leads to the bust.

The wave pattern reflects the inelasticity of demand and supply in the private equity market. Since the demand for private equity arises from exogenous shocks, it does not respond positively to supply. On the contrary, increases in supply reduce the demand faster. The supply of private equity is inelastic because the private equity firms only learn gradually about the profitability of investing. The *speed of learning* depends on the degree of investment specificity and on the market's prior beliefs. The more idiosyncratic a target company is, the less informative is its reorganization outcome about the prospects of reorganizing other companies. Furthermore, if the market ex ante perceives a high shock as very unlikely, it is more reluctant to interpret successful outcomes as a sign of general profitability. The *speed of entry* depends on the skill distribution among private equity firms. For instance, a skill pyramid with "few at the top, and many at the bottom" produces few entrants when expectations are low but many entrants when expectations are high. The combination of slow learning with a skill pyramid leads to waves with slow starts, explosive booms and sudden crashes.

When the market becomes more confident about the expected reorganization value, potential target companies increase in value, which in turn affects the negotiations between funds and target shareholders. Thus, a rise in market confidence not only attracts more private equity funds to the market but also raises the price that these funds must pay to acquire target companies. In other words, aggregate fund activity and valuation levels are jointly determined by market expectations and hence move together, consistent with the evidence in Kaplan and Stein (1993) and Gompers and Lerner (2000). However, even when expectations increase, the *true* profitability remains unaffected by learning. That is, higher valuation levels do not imply that investments are more profitable. In fact, as valuations increase relative to "fundamentals", average fund profitability declines during a wave. This decline is reinforced by the entry of less skilled private equity firms. Similarly, the model yields a rationale for the positive relationship between entry and past industry performance, and for the negative relationship between entry and subsequent fund performance, documented by Kaplan and Schoar (2005). High industry performance today raises market confidence, which increases fund activity tomorrow and—at the same time—decreases future fund performance via higher prices.

At the fund level, the heterogeneity among private equity firms immediately implies persistent differences in fund performance: a fund that has outperformed the industry is likely to continue to outperform the industry with its follow-on funds. A more interesting prediction of the model is that a private equity firm's time of entry is related to its quality. In any period, only the private equity firms above a certain threshold quality raise a fund, and the threshold is decreasing in the level of market confidence. Dynamically, this means that entry and exit follow a last-in-first-out pattern: As the level of market confidence varies over time, the least skilled private equity firms are always the latest to enter and, by the same token, always the first to exit the market. Thus, at any point in time, the first-time funds (the latest entrants) underperform the industry. However, their follow-on funds—if the boom continues—improve in relative performance as private equity firms of even lower quality will enter the market. The lowest-quality firms enter after highly profitable periods, when valuation levels are high, and during periods in which fund activity will ex post turn out to have peaked. Due to the last-in-first-out pattern, such firms are likely to exit the market soon after. Or putting it differently, funds first raised in boom times are less likely to see follow-on funds. These predictions are consistent with the evidence in Kaplan and Schoar (2005).

Kaplan and Schoar (2005) also study the relation between fund size and fund profitability and report that the relationship is—on the one hand—positive and concave across different funds, and—on the other hand— negative across consecutive funds of the same private equity firm. While the baseline model assumes a uniform and constant fund size, these patterns naturally arise in an extension that allows private equity firms to run larger funds at an increasing marginal cost. The firms' marginal cost functions reflect their management ability. For any given level of market confidence, cross-sectional variation in size is driven by variation in ability: larger funds are managed by better private equity firms, which is the reason why they are more profitable. By contrast, for a given firm (quality), time variation in fund size is driven by time variation in market confidence, i.e., purely by learning. When market confidence is higher, a private equity fund makes more acquisitions. At the same time, the fund pays higher prices (due to increased valuation levels) and operates at a higher average cost (due to its larger size). Thus, as the true profitability of investing is time-invariant, the fund's true expected profit (per investment) is inversely related to its size during a wave.

Finally, we study the effects of fund competition in a simple model extension which incorporates search frictions into the private equity market. In the presence of such frictions, a fund's bargaining power vis-à-vis a target company is weaker when there are more competing funds or fewer target companies in the market. This reinforces the link between market confidence and acquisition prices: when the market becomes more confident, the prices rise not only because a target's total expected reorganization value increases but also because the entry of new funds shifts bargaining power to the targets. Compared to the absence of competition, fund profitability drops faster as a result of fund entry or target attrition, i.e., when "more money chases fewer deals." Such congestion effects slow down entry and precipitate exit so that fund activity both builds up and declines more gradually than in the basic model. Thus, fund competition affects neither the boom-bust pattern nor the last-in-first-out pattern of fund activity, but it "smoothes" the wave.

The phenomenon of waves has previously been analyzed theoretically by Jovanovic and Rousseau (2002), Shleifer and Vishny (2002) and Rhodes-Kropf and Robinson (2008) in the context of mergers and acquisitions; and by Inderst and Müller (2004) and Michelacci and Suarez (2004) in the context of venture capital markets. These papers address neither the role of learning and attrition nor the endogenous *intra*-wave dynamics of investment, prices, and returns.

We are not the first to study the impact of learning on financial decisions. For instance, learning models have been used to explain financial innovations (Persons and Warther, 1997), stock market prices (Timmermann, 1993, 1996; Veronesi, 1999; Pastor and Veronesi, 2008), going public decisions (Pastor et al., 2006; He, 2007), and business cycles (Veldkamp, 2005; Van Nieuwerburgh and Veldkamp, 2006). Contemporaneous work by Hochberg et al. (2008) and Glode and Green (2008) also incorporates learning into a model of the private equity market. In both models, fund investors (limited partners) learn about the ability of fund managers (general partners). By contrast, in our model, fund managers learn about market conditions which affect the profitability of private equity investments.

The remainder of the paper is organized as follows. Section 2 presents the basic model. Section 3 derives the competitive Markov equilibrium. Section 4 analyzes the equilibrium dynamics. Section 5 presents the model extensions which incorporate fund size and fund competition. Section 6 concludes the paper.

2 Model

Consider a risk-neutral economy in discrete time, $t \in \mathbb{Z}_0^+$, with a fixed population of $\mathcal{N}$ companies. Initially, each company is run by an incumbent manager, and its discounted dividend value under the incumbent manager is normalized to 0.

In period 0, the economy experiences a latent productivity shock. The shock makes each company—if appropriately reorganized—improvable. A company's value after reorganization, V, is gamma-distributed with shape parameter $\alpha > 0$ and scale parameter $1/\beta > 0$. The mean of the gamma distribution, $\overline{V} = \alpha/\beta$, reflects the expected reorganization value. We assume that α is commonly known, whereas β is unobserved. Since a lower β translate into a higher expected reorganization value, this implies that the market is uncertain about the magnitude of the shock. The market's initial beliefs about β are also represented by a gamma distribution, with known shape and inverse scale parameters $\tau > 0$ and $1/\gamma > 0$ respectively.[3]

We assume that the incumbent managers cannot generate the value improvement. We also abstract from the possibility that they procure the necessary human capital through consulting services or the labor market. Instead, let there be $\mathcal{M}$ outside management teams who can carry out this task provided that they make a control investment in the company and set up the necessary operations. We henceforth refer to these potential investor-managers as private equity firms.[4]

In every period $t \geq 1$, each private equity *firm* decides whether or not to enter the market for corporate control for the duration of that period. To enter, the firm must raise and operate a fund which imposes a per-period cost (e.g., due to search activities, due diligence, negotiations, legal expenses). The cost is fixed but varies across private equity firms: $C_1 < C_2 < \cdots < C_{\mathcal{M}}$. For later use, we define a continuously increasing function $C(\cdot)$ with $C(i) = C_i$ for all $i \in \{1, 2, \ldots, \mathcal{M}\}$. This function reflects the talent distribution among private equity managers and is commonly known. To ensure interior equilibria, let $C(1) = 0$ and $C(\mathcal{M}) = \infty$.[5]

A private equity *fund* seeks to invest in companies. We assume that (human) resource or time constraints impose a limit on the number of investments that a fund can undertake simultaneously. To keep matters simple, we normalize this limit to one company per period. (Endogenous limits are discussed in section 5.1.) In every period, each active fund is paired with a potential target (or portfolio) company. Once paired, they negotiate the price at which the fund can purchase (a control stake in) the company. Negotiations are modeled as Nash

[3] For our purposes, the gamma distribution is attractive because it rules out negative value improvements and allows for a tractable Baysian analysis. The qualitative results should carry over to any stochastic setting with parameter uncertainty where high realizatons lead Bayesian agents to increase their expectations about the mean of the underlying probability distribution.

[4] Private equity funds often enforce changes in the governance of their portfolio firms (Gertner and Kaplan, 1996; Acharya and Kehoe, 2008; Cornelli and Karakas, 2008). Acharya and Kehoe (2008) report that one-third of CEOs in buyout targets are fired in the first 100 days.

[5] The formulation of heterogeneity in terms of cost is not to be taken too literally. Similar results obtain when private equity firms instead differ in their ability to improve their portfolio companies. We choose the cost formulation because it makes the analysis more tractable.

bargaining with $\omega \in (0,1)$ denoting the relative bargaining power of the fund. If a negotiation fails, the involved parties part and neither is paired again in the ongoing period. Otherwise, the fund purchases and reorganizes the company. A reorganized company harbors no further potential for improvement. Thus, there is attrition.

$M_t \leq \mathcal{M}$ and $N_t \leq \mathcal{N}$ respectively denote the number of private equity funds (operated) and potential target companies (available) *in* period t. For $M_t > N_t$, we adopt the convention that the most efficient funds are paired with a company first. Similarly, for $M_t < N_t$, we adopt the convention that those companies which have been in negotiations previously are paired with a fund first.

The timing of the model is as follows. In period 0, everyone in the economy learns about the occurrence of the shock but does not observe its magnitude, i.e. β. In each subsequent period $t \geq 1$, events unfold in the below order:

1. Everyone enters the period with beliefs $\overline{V}_t = E_t(\overline{V})$.
2. All private equity firms decide whether to raise a fund for the current period.
3. Funds are paired with a target company and bargain over the purchase price.
4. Funds that have successfully negotiated the price acquire their targets.
5. Acquired companies are reorganized and their new value becomes public.
6. Everyone updates their beliefs.

3 Equilibrium

The key decisions in the model are the private equity firms' repeated decisions of whether or not to raise a fund. Let $a_t^i \in \{1, 0\}$ denote firm i's decision in period t, where $a_t^i = 1$ if the firm decides to raise a fund, and $a_t \equiv (a_t^1, \ldots, a_t^{\mathcal{M}})$. We assume competitive behavior and rational expectations. That is, each private equity firm ignores its own impact on *aggregate* variables but has unbiased expectations about (the evolution of) these variables.

In each period t, the history of all previous investment outcomes is commonly known. The history has a *direct* impact on the payoffs from t onward only through its impact on the state variables $\overline{V}_t$ and N_t. Given a state $(\overline{V}_t, N_t)$, firm i chooses a_t^i to maximize the sum of its discounted expected future per-period profits:

$$\Pi^i(a_t, \overline{V}_t, N_t) = \mathrm{E}_t \left[\sum\nolimits_{\tau=t}^{\infty} \delta^{\tau-t} \pi_\tau^i(a_\tau, \overline{V}_\tau, N_\tau) \left| \overline{V}_t, N_t \right. \right]$$

where $\pi_t^i(a_t, \overline{V}_t, N_t)$ is i's period-t profit, and $\delta \in [0, 1]$ is the discount factor.

Our analysis focuses on Markov strategies which depend on the history solely through the current state of the world (see Maskin and Tirole, 2001). In a Markov equilibrium, the optimal entry strategies and the equilibrium profits can therefore be written as $a_t^* = a_t(\overline{V}_t, N_t)$ and $\Pi^i(a_t^*, \overline{V}_t, N_t)$. Given optimal future behavior, this allows us to decompose $\Pi^i(a_t, \overline{V}_t, N_t)$ into the profit from the current period and a future "franchise" value:

$$\pi_t^i(a_t, \overline{V}_t, N_t) + \delta \mathrm{E}_t[\Pi^i(a_{t+1}^*, \overline{V}_{t+1}, N_{t+1}) \left| \overline{V}_t, N_t \right.].$$

Importantly, i's decision today affects the future only through its impact on the aggregate state variables $\overline{V}_{t+1}$ and N_{t+1}. Under competitive behavior, each private equity firm ignores this (intertemporal) impact. Consequently, the firm treats the entry decisions in different periods like *independent* options—behaving de facto *as if* it were myopic. Intuitively, the private equity firm perceives the impact of its current investment on future market conditions as so small that its sole decision criterion is the immediate profit.[6]

The dynamics of the competitive Markov equilibrium are the focus of the present paper. The key driver of these dynamics is a feedback loop between entry decisions and market conditions: entry today depends on how market conditions have evolved, which in turn depends on past entry decisions. Therefore, we subsequently analyze entry decisions for given market conditions, and in turn market conditions for a given history of entry decisions.

[6]The assumption of competitive behavior has two principal consequences: On the one hand, firms with negative expected current profits do not take into account the possibility of active *experimentation*. As a result, they become *adaptive* learners (Van Nieuwerburgh and Veldkamp, 2004; Veldkamp, 2004). On the other hand, firms with positive expected current profits neglect the possibility of *procrastinating* entry to learn more from information produced by others.

3.1 Entry decisions

To determine entry in period t for a given state $(\overline{V}_t, N_t)$, we must first determine the outcome of the ensuing bargaining stage. Let P_t^i denote the purchase price that the fund (of firm) i and its potential target company bargain over. Furthermore, let O_t^i and O_t^c respectively denote the outside options (threat points) of the fund and the company. The Nash bargaining solution is given by

$$P_t^i = \arg\max(\overline{V}_t - P_t^i - O_t^i)^{\omega}(P_t - O_t^c)^{1-\omega}. \tag{1}$$

To derive the bargaining solution, we need to specify the outside options. For the quasi-myopic fund, the outside option is to save the amount—rather than to invest it in the company—for one period at the risk-free rate, which yields P_t/δ. Its current outside option is today's net present value of saving the amount, which is $O_t^i = \delta\,(P_t/\delta) - P_t = 0$. By contrast, the target company's outside option is the expected payoff from returning to the market in the hope of being acquired in the future. Suppose that a company which has been in negotiations previously is certainly paired with a fund in the next period. (This holds in equilibrium: a unilateral deviator would be the only such company, and would hence be paired with fund 1 in the next period). As in the literature on search markets, a deviator's payoff from a future match is the payoff from a successful deal, i.e., the future "inside" option. Thus, the company's current outside option is $O_t^c = \delta E_t[P_{t+1}]$.

Given these outside options, the Nash bargaining solution is $P_t = (1-\omega)\,\overline{V}_t + \omega\delta E_t[P_{t+1}]$. To get a closed-form solution, we conjecture an equilibrium outcome in which the price is a *linear* function of $\overline{V}_t$ such that $P_t = \psi\overline{V}_t$. Since $E_t[\overline{V}_{t+1}] = \overline{V}_t$ (by the Law of Iterated Expectations), it then follows that $E_t[P_{t+1}] = E_t[\psi\overline{V}_{t+1}] = \psi\overline{V}_t = P_t$. Thus, *if* P_t is a linear function of $\overline{V}_t$, it is a martingale. Conversely, if P_t is a martingale, the Nash bargaining solution is indeed linear in $\overline{V}_t$: substituting $E_t[P_{t+1}] = P_t$ into the bargaining solution yields

$$P_t = \frac{1-\omega}{1-\omega\delta}\overline{V}_t. \tag{2}$$

Thus, $\psi = \frac{1-\omega}{1-\omega\delta)}$ is a rational equilibrium outcome. Consistent with intuition, a more patient firm (lower δ) bargains for a higher price ($\partial\psi/\partial\delta > 0$). Furthermore, since a failure to agree is inefficient, all negotiations lead to a transaction.

Having derived the bargaining solution, we now turn to the entry decision. A quasi-myopic private equity firm raises a fund (only) if the current expected profit from investing is positive. That is, the firm enters the market if $C_i \leq \overline{V}_t - P_t = (1-\psi)\overline{V}_t$ and is sure to be matched with a target company. Since this is true for all private equity firms, there exists a cut-off cost C_{i^*} such that all and only firms with $C_i \leq C_{i^*}$ raise a fund. In fact, i^* is equivalent to M_t, the total number of funds raised in t. It is defined by $C(i^*) = (1-\psi)\overline{V}_t$ as long as $i^* < N_t$; and by $i^* = N_t$ otherwise.

Lemma 1 *There exists a competitive Markov equilibrium in which all and only private equity firms with $C_i \leq C(M_t) = \min\{(1-\psi)\overline{V}_t, C(N_t)\}$ enter the market for corporate control with a fund in period t. The number of funds M_t is increasing in $\overline{V}_t$ but decreasing in N_t, while the acquisition price P_t is increasing in $\overline{V}_t$.*

The equilibrium is intuitive: More talented private equity managers are more inclined to enter so that, in every period, the relatively "best" private equity firms raise a fund. Furthermore, more private equity funds are raised when the expected reorganization value is higher (or the funds have more bargaining power). The number of funds is also (weakly) increasing in the target stock N_t, i.e., the number of available target companies. Though the target stock only matters when it becomes a binding constraint ($N_t \leq M_t$). In section 5.2, we discuss possible channels for market congestion, which can cause the attrition in the target stock to have a more continuous impact on fund activity.

3.2 Market conditions

Lemma 1 characterizes the equilibrium outcome for a given state process $\{\overline{V}_t, N_t\}$. We now turn to the determination of this process. The target stock N_t monotonically decreases as more and more investments are completed. More specifically, if M^t denotes the number of investments consummated prior to t, the target stock at the beginning of period t is $N_t = \mathcal{N} - M^t$.

Past investment also allows market participants to make inference about the true β, i.e., to learn about the magnitude of the shock. In this respect, the revenue generated by each reorganization represents a noisy signal about $\overline{V}$. We assume that reorganization revenues are observable to other market participants. This assumption is not to be taken literally, since private equity firms are in practice known to be secretive about their returns. Rather, it parsimoniously captures the notion that information about superior profitability leaks—at least informally—to other potential targets or to investors who are interested in starting their own private equity funds. The information spillover is central to the dynamics, as it creates a intertemporal link between past performance and future market entry.

Let v_j denote the revenue generated by investment j. A history of investment outcomes is $\mathcal{H}_t = \{v_j\}_{j=1}^{M^t}$, and the historic average is $\overline{v}^t = \sum_{j=1}^{M^t} (v_j/M^t)$. Given a history $\mathcal{H}_t$, the posterior distribution of $\overline{V}$ is *inverse* gamma with shape and scale parameters $\tau + M^t\alpha$ and $\alpha(\gamma + M^t\overline{v}^t)$ respectively. (Details of the Bayesian updating process are provided in Appendix A.) In period t, the market's expectations about the reorganization value are equal to the mean of the inverse gamma distribution, $\overline{V}_t = E(V\,|\mathcal{H}_t)$, or more precisely

$$\overline{V}_t = \frac{\alpha(\gamma + M^t\overline{v}^t)}{\tau + M^t\alpha - 1}. \tag{3}$$

The conditional expectation (3) contains all distributional parameters except β, about which inference is being made. Recall that α is the known shape parameter of the V-distribution, whereas τ and $1/\gamma$ are the parameters of the distribution representing the market's initial (period-0) beliefs about the true β.

Lemma 2 $\overline{V}_t$ *is ceteris paribus (i) increasing in* $\overline{v}^t$*, (ii) increasing in* M^t *if and only if* $\overline{v}^t \geq \alpha\gamma/(\tau - 1)$*, and (iii) increasing in* α *and* γ *but decreasing in* τ*.*

Current expectations increase with the historic average, because good past outcomes indicate that the reorganization value is high. In addition, if the historic average is high (low) relative to initial expectations, current expectations increase (decrease) in the number of past investments. The reason is that additional observations increase the precision of the estimate (in either direction). Finally, current expectations are higher when the initial expectations $\overline{V}_0 = E(\alpha/\beta\,|\mathcal{H}_0)$ were high, which explains why they are increasing in α and decreasing in $E(\beta) = \tau/\gamma$. In the subsequent analysis, we assume that $\overline{V}_0$ is strictly positive, though very small. This is meant to capture that, absent positive experiences, the market is sceptical about the prospects of reorganization.

4 Dynamics

We now study entire equilibrium paths to characterize the dynamics of aggregate fund activity, prices and returns. A conceptual difficulty is that, even for a given β, the economy evolves stochastically so that there is no *unique* equilibrium path. To describe "typical" properties of an equilibrium path, we characterize the path that is obtained when every reorganization yields the mean revenue $\overline{V}$. We refer to this path (somewhat incorrectly) as the "trend" path, and index it with o.

It is important to bear in mind that the agents in the model are unaware that the deviations from the mean are zero on the trend path. Hence, they update their beliefs as if the reorganization revenues were genuinely random. More precisely, since $\overline{v}^t = \overline{V}$ for all t, market expectations on the trend path evolve according to

$$\overline{V}_t^o = \frac{\alpha(\gamma + M^t \overline{V})}{\tau + M^t \alpha - 1}. \tag{4}$$

The expectations monotonically converge to $\overline{V}$ as M^t goes to infinity. The speed of convergence decreases for large absolute values of τ and γ (keeping their ratio constant). Accordingly, one may interpret a large value of $\tau = \bar{z}\gamma$ for constant $\bar{z}$ as a low "signal-to-noise" ratio.

4.1 Waves

In $t = 0$, the economy receives news about the occurrence of the shock and forms prior expectations about the expected reorganization value. For entry to occur, these expectations must exceed $C_1/(1 - \psi)$ so that at least private equity firm 1 finds it worthwhile to raise a fund (Lemma 1). Since $C_1 = 0 < \overline{V}_0$, there is initial entry and consequently some learning that can serve as impetus for future entry.

4.1.1 Learning and attrition

Given entry, the evolution of fund activity (on the trend path) is determined by the true $\overline{V}$. If $\overline{V}$ is small, the initial reorganizations generate modest revenues, and investment activity remains low. Indeed, for $\overline{V} < \overline{V}_0$, the revenues disappoint the market and investment activity subsides. By contrast, if $\overline{V}$ is large, the

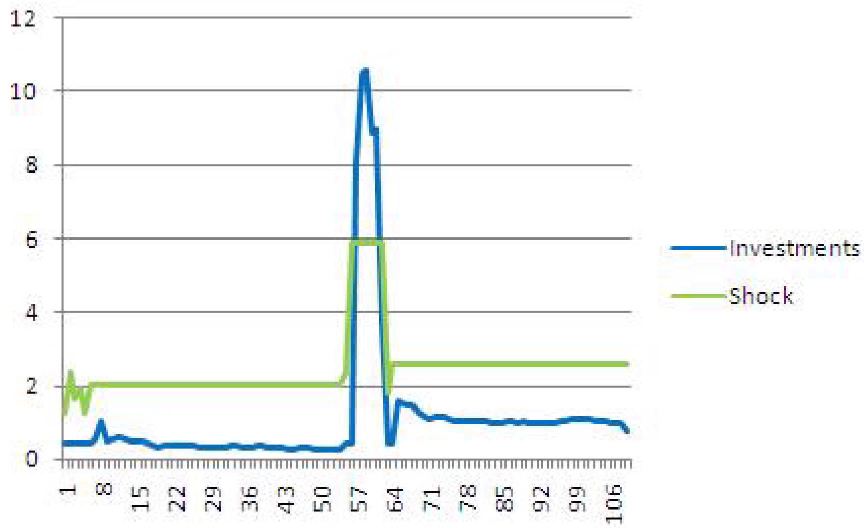

Figure 1: Long-run pattern

market becomes increasingly optimistic because the investments are more profitable than expected. This attracts new funds, which in turn causes the target stock to decline faster. The two effects, *learning* and *attrition*, have countervailing consequences for future fund activity. When the number of funds reaches the number of remaining targets, investment climaxes and then collapses.

The ultimate decline in investments is rather extreme on the trend path. Yet, it epitomizes the wave pattern inherent in any equilibrium path. Even on stochastic paths, investment booms endogenously transition to sudden busts.

Proposition 1 *Expansions in fund activity follow a boom-bust pattern.*

In reality, productivity shocks occur more than once. In most cases, the shocks are probably small with little impact on overall activity. In a few cases, however, the shocks may be large, leading to a wave-like expansion in fund activity. While ex post observed, such waves are ex ante unpredictable. To illustrate such a long-run pattern, we simulate the equilibrium paths for a large number of shocks $\{\beta_k\}$ drawn from a gamma distribution with a high mean τ/γ (so that $\overline{V}_0$ is low). Figure 1 depicts a representative sequence of shocks with the fund activity that followed in their wake. As expected, lengthy periods with little fund activity are interrupted by a rare large wave. Thus, the model can plausibly produce patterns that are consistent with the documented cyclicality

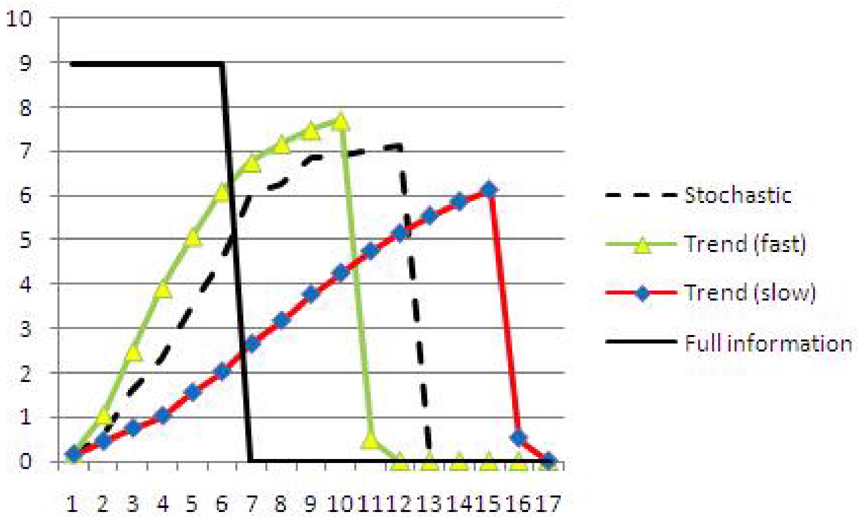

Figure 2: Different speeds of learning.

of private equity activity (Kaplan and Stein, 1993; Lerner, 2002; Acharya et al., 2007; Kaplan and Strömberg, 2008).[7]

4.1.2 Inelastic supply

The specific shape of a wave depends on the speed of entry, which in turn depends on the speed of learning and on the skill distribution $C(\cdot)$. On the one hand, when learning is slow (high $\tau = \bar{z}\gamma$), the market develops confidence more slowly. On the other hand, when skill is scarce (high $C' > 0$ and low $C'' < 0$), private equity firms want to be more confident before they enter. When slow learning and skill scarcity are combined, fund activity incubates slowly, then suddenly booms, and crashes in the end. The boom occurs when market confidence reaches a level that attracts many entrants, which in turn accelerate learning and boosts confidence even further. The crash occurs because, once fund activity reaches its climax, the high rate of attrition rapidly diminishes the target stock. (The magnitude of the wave depends, of course, also on the true $\overline{V}$ and on the initial target stock $\mathcal{N}$.)

Figure 2 depicts four different equilibrium paths following a large shock

[7] For instance, venture capital activity expanded during the biotechnology boom in the early 1990s and during the information technology boom in the late 1990s. Similarly, buyout activity experienced high levels in the 1980s and in the mid-2000s.

($\overline{V} \gg \overline{V}_0$). The plain solid line is the equilibrium path when $\overline{V}$ is immediately observed. The other two solid lines (marked with triangles and diamonds respectively) are trend paths that differ in the speed of learning. Finally, the dashed line depicts the stochastic path that corresponds to the trend path with faster learning. Comparing the different paths bears on the notion of inelastic supply and demand in the private equity market (Gompers and Lerner, 1999). Demand inelasticity is hard-wired into the model. Demand arises due to the exogenous productivity shock; as such, it does not respond to changes in supply. By contrast, supply inelasticity is endogenous. Supply responds slowly to changes in demand because private equity managers do not enter until they are confident enough. Accordingly, supply is less elastic when learning is slower or skill is scarcer.

4.2 Industry

We now describe in more detail how, at the industry level, (i) fund activity relates to valuation levels in the market, (ii) average fund performance evolves during a wave, and (iii) fund activity relates to past and future performance.

4.2.1 Entry and valuation

When the market grows more confident about the expected reorganization value, potential target companies increase in value. That is, a rise in market confidence not only attracts more private equity funds to the market but also raises the price that these funds must pay to acquire target companies (Lemma 1).

Proposition 2 *Fund activity and valuation levels increase together.*

Proposition 2 is consistent with Kaplan and Stein (1993) who document that, during the buyout wave in the 1980s, buyout prices rose relative to fundamentals. Gompers and Lerner (2000) find similar results using a large data set comprising private equity investments in different stages and industries from 1987 to 1995. Specifically, they report that capital inflows into the private equity industry coincided with higher valuations of the funds' new investments. Both papers argue that the valuation increases were driven by fund competition rather than by improved investment prospects, suggesting that too much capital was chasing too few attractive investment opportunities.

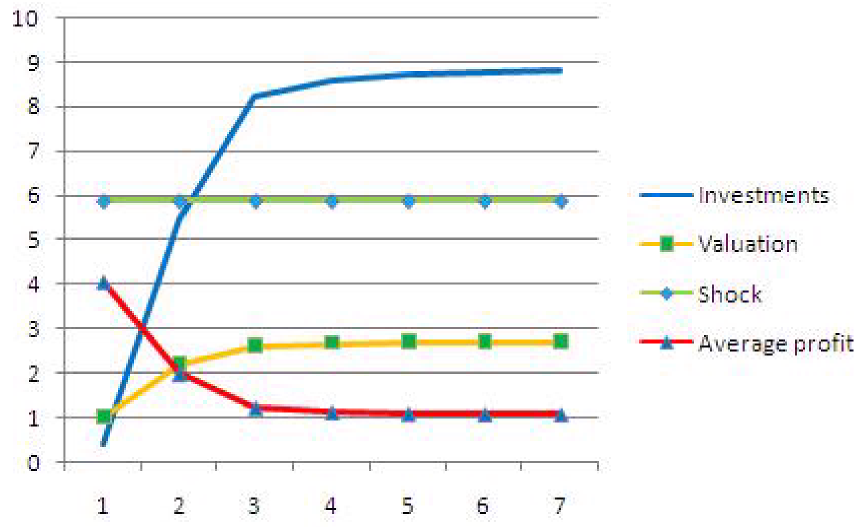

Figure 3: Entry, valuation and average profit.

The model can explain the observed pattern even in the absence of fund competition (which we introduce in section 5.2). Higher entry and higher valuations are *jointly* caused by learning about the expected reorganization value. However, neither effect coincides with a concurrent or subsequent increase in the actual reorganization value.[8] The top three lines in Figure 3 illustrate these relationships for a trend path.

4.2.2 Cross-sectional average performance

In spite of learning, the market expectations $\overline{V}_t$ typically diverge from the true $\overline{V}$. When taking the model to empirical data, this distinction is crucial as observed fund revenues reflect the true $\overline{V}$—as opposed to the *subjective* expectations $\overline{V}_t$. Model predictions about fund performance therefore depend on $\overline{V}$. At the industry level, the *true* (data-generating) process that determines *average* per-period fund profits is $\bar{\pi}_t = \overline{v}_t - P_t - \overline{C}_t$, where $\overline{v}_t$ is the average gross return (reorganization revenue), and $\overline{C}_t = \sum_{i=1}^{M_t} (C_i/M_t)$ reflects average fund quality, in period t.

To see how average per-period fund profits evolve on the trend path, we simply need to set $\overline{v}_t = \overline{V}$, $P_t = P_t^o = \psi \overline{V}^o_t$, $M_t = M_t^o$, and $\overline{C}_t = \overline{C}_t^o =$

[8] If the shock to profitability is a shock to future cash flows, the increase in valuation levels corresponds to an increase in valuation multiples, such as the price-earnings ratio.

$\sum_{i=1}^{M_t^o} (C_i/M_t^o)$. For $\overline{V} > \overline{V}_0$, which induces a wave, we know from (4) that market confidence, $\overline{V}_t^o$, monotonically increases over time. This causes prices, P_t^o, and fund activity, M_t^o, to monotonically increase (Lemma 1) but average fund quality, $1/\overline{C}_t^o$, to monotonically decrease (see section 4.3.1). All the while, the *true* expected reorganization value, $\overline{V}$, remains *constant.* The rising prices and the declining quality thus imply that $\bar{\pi}_t^o$ decreases over time.

Proposition 3 *In a wave, average per-period fund performance tends to decrease.*

The line marked with triangles in figure 3 shows the evolution of average fund profits on a trend path. The decrease in average profits is steeper than the increase in prices because of the declining fund quality. It is noteworthy that Proposition 3 is not the result of increased fund competition. It merely requires learning and heterogeneity among private equity firms.

The decline in fund profitability across vintages appears to be at odds with the empirical finding that first-time funds underperform the industry (Kaplan and Schoar, 2005). However, this is not the case if the comparison between first-time and later-time funds is made in the cross-section, or if the comparison between first-time and later-time funds by the same private equity firm is based on the performance *relative* to the industry. Sections 4.3.2 and 4.3.3 elaborate on these points. Nevertheless, the model cannot explain *systematic* increases in the *absolute* performance of consecutive funds by the same private equity firm during a wave.

4.2.3 Lagged entry-performance correlations

Kaplan and Schoar (2005) find that capital flows *to* the private equity industry are positively correlated with last period's industry returns but negatively correlated with next period's fund returns. Note that our model in general exhibits dynamics where industry growth goes together with a decline in fund profits, i.e., where high past performance precedes high(er) future entry and low(er) future performance.

To highlight such dynamics, let us consider a stochastic path for a shock that happens to coincide with the market's initial expectations, $\overline{V} = \overline{V}_0$. The dynamics on the stochastic path are driven by the exogenous random process $\{\overline{v}_t\}$, i.e., the random (average) per-period revenues. As market confidence in the next period, $\overline{V}_{t+1}$, positively depends on the average revenues in the current period, $\overline{v}_t$, the process $\{\overline{v}_t\}$ serves as a "leading" indicator. To see this, note that

$$\overline{v}^t = \frac{M^{t-1}}{M^t}\overline{v}^{t-1} + \frac{M_t}{M^t}\overline{v}_t \quad \text{and} \quad \overline{V}_{t+1} = \frac{\alpha\left(\gamma + M^t \overline{v}^t\right)}{\tau + M^t \alpha - 1}$$

and therefore

$$M_{t+1} = C^{-1}\left[\left(1-\psi\right)\overline{V}_{t+1}\right], \quad P_{t+1} = \psi \overline{V}_{t+1}, \quad \text{and} \quad \overline{C}_{t+1} = \sum_{i=1}^{M_{t+1}} \left(C_i / M_{t+1}\right)$$

are all increasing in $\overline{v}_t$. That is, via the historic average, high period-t revenues increase market confidence, fund activity, prices, and average costs in period $t+1$.

Now consider the preditive power of $\{\overline{v}_t\}$ with respect to fund i's per-period fund profits $\{\pi_t^i\}$. Given a history up to t, the mean of the true (data-generating) distribution of i's profit in $t+1$ is

$$E_{t+1}\left[\pi_{t+1}^i \left| \overline{V}_{t+1} = \overline{V}_0 \right.\right] = \overline{V}_0 - \psi \frac{\alpha\left(\gamma + M^t \overline{v}^t\right)}{\tau + M^t \alpha - 1} - C_i,$$

which increases in the average period-t revenue $\overline{v}_t$ (via the historic average $\overline{v}^t$). Similarly, consider $\Delta\left(\overline{v}_t\right) \equiv E_{t+1}\left[\overline{\pi}_{t+1} \left| \overline{V}_{t+1} = \overline{V}_0 \right.\right] - \overline{\pi}(t)$, which represents the expected "drop" in average fund profits from t to $t+1$ as a function of $\overline{v}_t$:

$$\Delta\left(\overline{v}_t\right) = \overline{V}_0 - \overline{v}_t + \left(P_t - P_{t+1}\right) + \left(\overline{C}_t - \overline{C}_{t+1}\right).$$

Clearly, $\Delta'\left(\overline{v}_t\right) = -1 - \left(\partial P_{t+1} / \partial \overline{v}_t\right) - \left(\partial \overline{C}_{t+1} / \partial \overline{v}_t\right) < 0$. On the one hand, a higher average revenue today both increases prices ($\partial P_{t+1} / \partial \overline{v}_t > 0$) and decreases average fund quality ($\partial \overline{C}_{t+1} / \partial \overline{v}_t > 0$) tomorrow. On the other hand, since average per-period revenues, $\{\overline{v}_t\}$, are independent draws from distributions with mean $\overline{V}_0$, any realization $\overline{v}_t > \overline{V}_0$ means that the market was "lucky" in t. In comparison, the revenues in $t+1$ are likely to be "corrected" downwards.

Proposition 4 *High industry performance predicts high entry, which in turn predicts lower industry performance.*

Figure 4 illustrates the performance-entry patterns of a stochastic path. One may be tempted to view them as "bad timing" by private equity firms that choose to enter the market when profitability drops, while being absent when

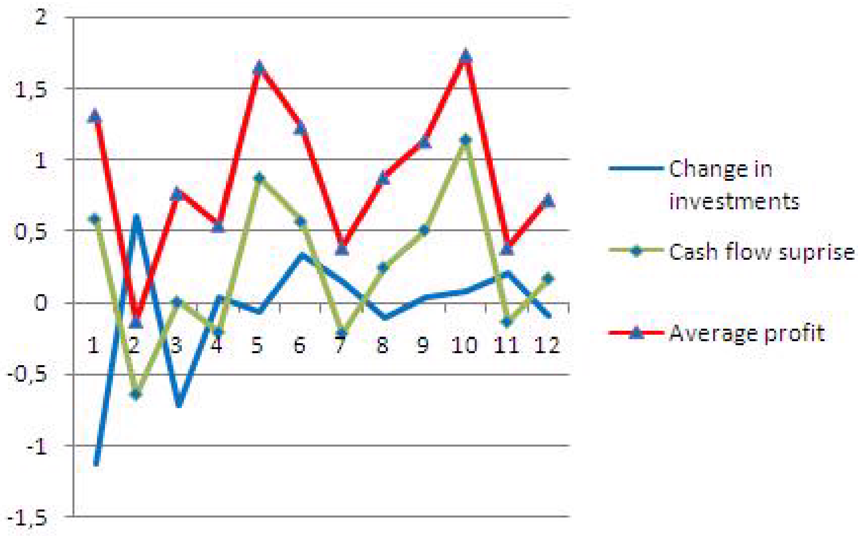

Figure 4: Lagged correlations.

profitability is high. However, such patterns emerge naturally in a model with learning, where changes in perceived profitability and in actual profitability do not necessarily go in the same direction.

4.3 Funds

Given skill heterogeneity and entry timing, the model also generates both cross-sectional and time-series predictions about performance at the level of individual funds, to which we turn below.

4.3.1 Persistent differences and last-in-first-out pattern

While average profitability declines during a wave, performance differences among private equity firms are persistent. That is, a firm (or a particular fund) that has outperformed the industry likely continues to outperform the industry in subsequent periods. This follows directly from the assumed skill heterogeneity, and is consistent with the empirical evidence (Kaplan and Schoar, 2005).

A more interesting implication of the model is that a private equity firm's quality and its time of entry are related. By Lemma 1, all and only firms above a threshold quality level C_{i^*} enter the market, and this threshold level is increasing in the expected reorganization value $\overline{V}_t$. This implies that, if the market becomes

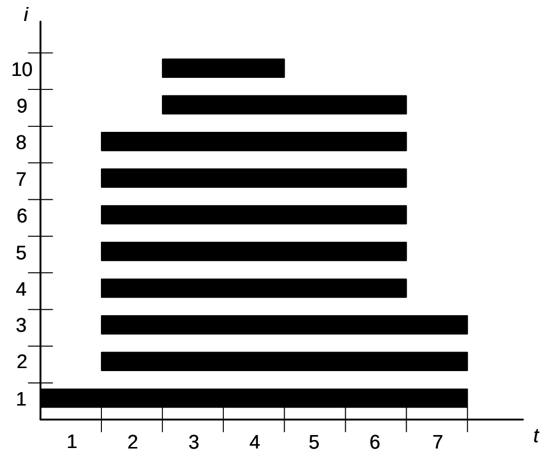

Figure 5: Last in, first out

more confident (higher $\overline{V}_t$), the funds raised by newly entering firms are of lower quality than the funds of "incumbent" firms. By the same token, if the market becomes less confident (lower $\overline{V}_t$), the firms that exit—i.e., do not raise a follow-up fund—are of lower quality than the firms that remain in the market. Thus, as $\overline{V}_t$ varies over time, entry and exit follow a last-in-first-out pattern: the least talented are the latest to enter when market conditions improve, and the earliest to exit when the conditions deteriorate. Figure 5 illustrates this for the case of ten partnerships and a stochastic path that lasted for seven periods.

4.3.2 First-time fund underperformance

The last-in-first-out pattern endogenously creates a cross-sectional link between a fund's "age" and its performance relative to the industry. For example, first-time funds are run by less skilled managers than contemporaneous later-time funds.

Proposition 5 *Funds with short track records tend to underperform the industry and are less likely to raise follow-on funds.*

Proposition 5 highlights that a positive relationship between the maturity of a private equity fund and its performance need not (solely) be driven by experience gains ("learning-by-doing"). Rather, it may reflect a causal relation between the fund managers' intrinsic abilities and their timing of entry and exit. Note further that many new funds are raised after highly profitable periods (Proposition 4),

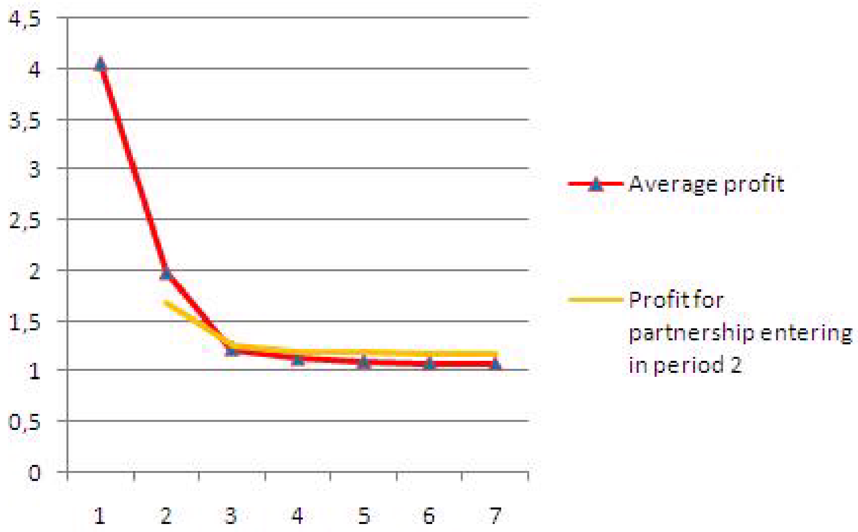

Figure 6: First-time funds.

when valuation levels are high (Proposition 2), and during periods in which fund activity ex post turns out to have peaked (Proposition 1). Given the last-in-first-out pattern, these funds are run by the least qualified managers who are likely to exit the market soon after.

Corollary 1 *Funds first raised in boom times are less likely to see follow-on funds.*

Proposition 5 and Corollary 1 are both consistent with the evidence in Kaplan and Schoar (2005). Broadly speaking, the predicted last-in-first-out pattern says that many "transient" private equity firms emerge during a wave, while the firms that are left at the end are those that have been around from the beginning.

4.3.3 Relative improvement over time

Proposition 5 is a potential explanation for why first-time funds (young private equity firms) underperform the industry in the cross-section. At the time a private equity firm i enters the market with its first fund, it belongs to the least skilled firms in the industry. However, if the boom continues, even less skilled firms enter in subsequent periods. As a result, the relative quality of firm i's follow-on funds improves over time.

Proposition 6 *Consecutive funds tend to improve in relative performance.*

Figure 6 illustrates this result by comparing the average fund profit on a trend path with the profit of a private equity firm that enters the market in period 2. While its first fund performs below average, its follow-on funds outperform the average fund in the industry from period 3 onwards.

5 Extensions

5.1 Fund size

Kaplan and Schoar (2005) also study the relationship between fund size and fund profitability and report two distinct findings: the relationship is positive and concave across different funds, whereas it is negative across funds from the same private equity firms. Our baseline model is mute on this issue as it assumes a uniform and constant fund size. In this section, we extend the model to allow for variable fund size and show that the above relationships between size and profitability arise naturally.

For simplicity, suppose that $\mathcal{M} = 2$. Each private equity firm $i \in \mathcal{M}$ can now undertake as many investments as desired. However, we assume that a firm's per-period cost of operating a fund is increasing and convex in the number of considered investments. More specifically, let $C_{it}(M_{it}) = (M_{it} + C_i)^2$ where C_i is a constant that reflects the (inverse) quality of firm i, and M_{it} is the number of investments undertaken by firm i in period t.[9]

5.1.1 Fund size and cross-sectional performance

As long as $M_t \leq N_t$ is not a binding constraint, the number of investments chosen by private equity firm i in period t satisfies $C_{it}(M_{it}) = (1 - \psi)\overline{V}_t$. This yields

$$M_{it} = \sqrt{(1 - \psi)\overline{V}_t} - C_i.$$

Since $C_1 < C_2$, this immediately implies that the fund of firm 1 is larger than the fund of firm 2. That is, fund size increases with fund quality.

[9]The results also hold for $C_{it}(M_{it}) = M_{it}^2 + C_i$. In this case, a fund's marginal cost per investment is the same across all partnerships. By contrast, under the cost function in the text, a fund's marginal cost per investment decreases in the partnership's talent.

We measure a fund's profitability by its true expected profit *per investment*

$$\frac{M_{it}(\overline{V}-P_t)-C_{it}(M_{it})}{M_{it}}=\overline{V}-P_t-\frac{(1-\psi)\overline{V}_t}{\sqrt{(1-\psi)\overline{V}_t}-C_i}$$

which is decreasing in C_i. Thus, the larger fund earns a higher return per investment. The reason is that the average cost per investment is lower for the better fund, whereas the true expected revenue per investment $\overline{V}-P_t$ is the same for both funds. Rewriting the expected profit per investment as $\overline{V}-P_t-(1-\psi)\overline{V}_t/M_{it}$ and differentiating twice with respect to M_{it} furthermore shows that the relationship between fund size and fund profitability is concave.

Proposition 7 *Within the cross-section of funds, performance is increasing and concave in fund size.*

This is consistent with the first of the two findings mentioned above. For given market expectations, the better private equity firm raises a larger fund. Fund size and fund profitability are jointly driven by the fund managers' quality, and hence positive correlated. This result relies on the heterogeneity among fund managers but does not exploit the dynamic properties of the model, to which we turn next.

5.1.2 Fund size and time-series performance

To examine how a fund size and fund profitability evolve during a wave, consider two arbitrary points in time, t'' and t', such that $\overline{V}_{t''}>\overline{V}_{t'}$. From the above analysis, it follows that (as long as $M_t\leq N_t$ is not a binding constraint) a private equity firm i raises a larger fund in t'' than in t', i.e., $M_{it''}>M_{it'}$. Its true expected profit in t can be written as

$$\overline{V}-P_t-\frac{(M_{it}+C_i)^2}{M_{it}}.$$

Since $P_t=\psi\overline{V}_t$, we know that $P_{t''}>P_{t'}$. Furthermore, $(M_{it}+C_i)^2/M_{it}$ is increasing in M_{it}. Taken together, this implies that the true expected revenue per investment $\overline{V}-P_t$ is lower in t'' (due to the higher prices), while the average cost per investment is higher in t'' (due to the larger fund size).

Proposition 8 *Across consecutive funds of the same private equity firm, fund performance is decreasing in fund size.*

During a wave, market expectations tend to rise over time. Proposition 8 says that, as a result, private equity firms will raise larger but less profitable funds in the course of a wave. In fact, the decrease in profitability across consecutive funds will be proportional to the increase in size, consistent with the second finding by Kaplan and Schoar (2005).

5.2 Fund competition

One approach to modeling fund competition is to incorporate search frictions into the model. With search frictions, the more parties enter one side of the market, the more difficulty they have in finding alternative trading partners. As a result, bargaining power shifts to the other side of the market.[10]

Such "congestion" effects arising from fund competition tend to reinforce many of the conclusions of our model. To illustrate this, we split the bargaining game in stage 3 into three substages. In substage 3-1, each fund is paired with a company. As before, they bargain over the price at which the fund can acquire the company. Each pair that successfully negotiates the price moves immediately to stage 4. If a negotiation fails, the pair moves to substage 3-2, in which the fund tries to find another target company. The probability of finding a new target is given by the matching function $\phi(m, n)$, where m is the number of funds contemporaneously searching for a new target, and n is the number of available target companies. In substage 3-3, the fund bargains with a newfound target or, when the search fails, resumes negotiations with the previous one. In either case, successful negotiations lead to stage 4. A failure to agree moves the pair to the next period.

We make standard assumptions about the matching function: $\partial\phi/\partial m < 0$ and $\partial\phi/\partial n > 0$. For a fund, the probability of being matched with a (new) company is lower when there are many other funds on the search, and higher when there are many available target companies. To simplify matters, we further assume that the target companies have all the bargaining power in substage 3-3.

[10] Several papers have used this approach to model venture capital markets and merger markets (Inderst and Mueller, 2001; Michelacci and Suarez, 2004; Rhodes-Kropf and Robinson, 2008).

Let $\rho \in (0, \delta)$ denote the *intra*-period discount factor between substages 3-1 and 3-3.

We solve the bargaining game for an arbitrary fund i in period t by backward induction. In substage 3-3, any company negotiating with the fund offers the price $\overline{V}_t$, and the fund accepts the offer. In substage 3-1, the initial fund-company pair bargains under the conjecture that all contemporaneous negotiations are successful (which is true in equilibrium). Thus, the fund's and the company's outside options are given by $O_t^i = 0$ and $O_t^c = \rho \left[1 - \phi\left(1, N_t - M_t\right)\right] \overline{V}_t$. (The pool of alternative target companies excludes the $M_t - 1$ companies that are conjectured to successfully negotiate with the other funds and the current negotiation partner.) The Nash bargaining solution (1) is then given by $P_t = \psi \overline{V}_t$ where

$$\psi = (1 - \omega) + \omega\rho \left[1 - \phi\left(1, N_t - M_t\right)\right].$$

Given the properties of the matching function, the price is increasing in the number of funds and decreasing in the number of potential targets. In reduced form, we can therefore define the sharing rule as a function $\psi(N_t, M_t)$ where $\partial\psi/\partial N_t > 0$ and $\partial\psi/\partial M_t < 0$. Note that $\psi(N_t, M_t)$ measures the degree of fund competition. It is worth emphasizing that, along with N_t and M_t, the degree of competition endogenously varies over time. For example, by attracting more entry, an increase in market confidence, $\overline{V}_t$, will increase fund competition.

Thus, the key difference to the basic model is that the sharing rule ψ is not time-invariant but increases with entry and attrition. In a model with fund competition, prices therefore increase—and fund profitability deteriorates—faster as more funds enter the market and the target stock is depleted, capturing the idea that profits drop when "more money chases fewer deals." This slows down entry and precipitates exit so that the fund activity both builds up and declines more gradually than in the basic model. In other words, fund competition neither undermines the boom-bust pattern nor the last-in-first-out pattern of fund activity; it merely "smoothes" the wave.

6 Concluding Remarks

The paper presents a model of the private equity market in which heterogenous private equity firms learn about investment profitability from past outcomes and the stock of potential target companies is depletable. We derive the optimal entry and exit strategies of private equity firms as a function of their ability and market expectations. A characteristic feature of the model is that large expansions in private equity activity occur in waves with endogenous transitions from booms to busts. In addition, the model matches a wide range of stylized facts regarding the dynamics of investment, prices and performance *during* a wave.

Appendix A: Derivation of $\overline{V}_t$

Let X be a gamma-distributed random variable with shape parameter α and scale parameter θ. It is convenient to define $\beta = \theta^{-1}$ as the inverse scale parameter. The expected value of X is equal to $\alpha\theta$ or equivalently $\alpha\beta^{-1}$.

In Bayesian probability theory, a class of prior probability distributions $p(\zeta)$ is said to be conjugate to a class of likelihood functions $p(x|\zeta)$ if the posterior distributions $p(\zeta|x)$ belong to the same family as the prior probability distributions.

The gamma distribution is a conjugate prior to itself whenever the likelihood function is a gamma distribution with known shape parameter α and unknown inverse scale parameter β. Suppose that we have a random sample $\{x_i\}_{i=1}^{n}$ of a gamma-distributed random variable X with known shape parameter α and unknown inverse scale parameter β. The likelihood function is then a gamma distribution with known shape parameter α and unknown inverse scale parameter β. If the prior probability distribution for β is a gamma distribution with known shape and inverse scale parameters τ and γ respectively, then the posterior distribution is a gamma distribution and has a shape parameter equal to $\tau + n\alpha$ and an inverse scale parameter equal to $\gamma + \sum_{i=1}^{n} X_i$.

In addition, if a random variable X is gamma distributed with shape parameter α and scale parameter θ then the random variable X^{-1} is inverse gamma distributed with shape parameter α and scale parameter $\theta^{-1} = \beta$. The expected value of the random variable X^{-1} is then $\beta/(\alpha - 1)$. Finally if the random variable X is inverse gamma distributed with shape parameter α and scale parameter $\theta^{-1} = \beta$ then the random variable cX, where $c \in R^{+}$, is inverse gamma distributed with shape parameter α and scale parameter $c\theta^{-1} = c\beta$.

In our particular case, this is all we need to derive the conditional expectation of the magnitude of the shock. Simply let $\alpha \to \alpha$, $\beta \to \beta$, $\tau \to \tau$, $\gamma \to \gamma$, $n \to M^t$, $X_i \to v_j$ and it immediately follows that

$$\overline{V}_t = E(V\,|\mathcal{H}_t) = E(\alpha\beta^{-1}\,|\mathcal{H}_t) = \frac{\alpha\gamma + \alpha\sum_{i=1}^{M^t} v_j}{\tau + M^t\alpha - 1},$$

where $\mathcal{H}_t = \{v_j\}_{j=1}^{M^t}$.

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